Summer Research Internship

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Finite-time Distributed Coordination Control for Multiagent Systems

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Chapter 1

abstract

This paper presents a novel controller that yields finite-time stability for multi-agent systems without collisions. We first state the problem and our setup. Then we present the novel finite-time controller based on barrier function and the information from the neighbor agents. We present simulation results for this system, demonstrating some application like special configurations formed by the autonomous driving agents. Finally, we present the conclusion and future work, followed by personal contribution.

Chapter 2

Introduction

This paper presents a novel controller that yields finite-time stability for multi-agent systems. We first state the problem and our setup. Then we present the novel finite-time controller based on barrier function and the information from the neighbor agents. We present simulation results for this system, demonstrating some application like special configurations formed by the autonomous driving agents. Finally, we present the conclusion and future work, followed by personal contribution.

Multi-agent systems have attract attention during the past decade, partly due to the potential application in autonomous driving, exploring dangerous environment and other robot collaboration situations. Till recently, various control problems have been introduced, namely consensus(also named rendezvous), flocking, formation and distributed coordination. The interested reader is referred to [1]-[4].

In spite of focus on connectivity preservation, which is of high importance, finite time stability is not achieved. Finite Time Stability (FTS) is of essential importance because in real cases multiple agents need to one or multiple common goals within limited time. The author in [5] focus on continuous autonomous systems and present Lyapunov-like necessary and sufficient conditions for a systems to exhibit FTS. While in [6]-[9] consensus and formation control problems within finite time are well presented by several classes of protocols. And this article focuses on FTS based on the barrier function in [10]. We achieved finite-time distributed coordination control without collision for multi-agent system, while merely single agent is discussed in [10].

However, some disadvantages can been seen in our setup. Firstly, the agents are limited within a circle so that they can maintain connectivity throughout time. Secondly, to simplify the protocol, multiple static agents are placed on the circular boundary, which is difficult to achieve in real cases.Thirdly, the system may not achieve designed coordinates since there may exist equilibrium points except for the goal position for an agent. Because the velocity of the nearest agent is introduced, so the equilibrium points are tough to predict. This problem is discussed in the discussion part.

The paper is organized as follows: Chapter 3 presents the model and problem is stated. In chapter 4, we introduce the barrier function as well as the control law for collision avoidance and convergence to the goal positions within finite time . Simulation can be seen in chapter 5. In chapter 6, our conclusion and thoughts on future work are summarized . Finally, we present personal contribution in section 7.

Chapter 3

Modeling and Problem Statement

Consider a network of N mobile agents deployed in a known workspace W. Each agent $i \in \{1, ..., N\}$ is modeled as a circular disk of radius r and circular agents are centered at known positions $\vec{x_i}$, $i \in \{1, ..., N\}$. Moreover, we assume all kinetic agents are located within a bounded circle of radius R. On the boundary, there locate M static agents with radius of r. With collision avoidance of agents on the boundary, kinetic agents remain in the circle according to designed barrier function. As a result, network connectivity is preserved and one agent receives the position and velocity information of the other agents. Now consider one kinetic agent i. Its motion under single integrator dynamics is

$$\dot{\vec{x_i}} = \vec{v_i},$$

where $\vec{x_i}, \vec{v_i} \in \mathbf{R^n}$. The problem of reaching to a specified goal position in finite time can be formulated mathematically as follows:

$$\exists t* < \infty \; s.t \; \forall t > t* \; ||\vec{x_i}(t) - \vec{\tau_i}|| = 0,$$

where $\vec{\tau_i}$ is the desired goal location, while that of collision avoidance can be written as:

$$\forall t > t_0 \; ||\vec{x_i}(t) - \vec{x_j}(t)|| > d_c,$$

where $\vec{x_i}(t)$ and $\vec{x_j}(t)$ represent the location of the agents and t_0 is the starting time. Here,we model that d_c is bigger than $2r$. Also, we assume that the distances between goal positions $||\vec{\tau_i} - \vec{\tau_j}|| > d_c$. We also assume that one agent starts sufficiently far away from the other agents so that $||\vec{x_i}(t_0) - \vec{x_j}(t_0)|| > d_c$. What we should do is to design a feedback-law $\vec{v_i}$ such that all agents reach their own goal positions within finite time while maintaining safe distance between each other. Specifically speaking, we design a controller based on a barrier function and information of other agents.

Chapter 4

Motion Coordination

All the agents initiate in the closed region where the wireless communication links can be established. As the agents are moving inside the region during the process, their wireless communication can remain stable, thus resulting in connectivity maintenance. As mentioned in Section 3, for any one of the agents, for example the i^{th} agent $\vec{x_i}$, can not only know the location of the boundary but also perceive the position and velocity of its nearest agent, for example $\vec{x_j}$. We assume for now that:

1) The goal positions $\vec{\tau_i}, i \in \{1, 2, ..., n\}$ are static and,

2) There are no physical obstacles in the region and,

3) The distance between any two agents' goal positions $\|\vec{\tau_i} - \vec{\tau_j}\| > d_c$ and,

4) All n agents are equal, *i.e.*, there is no leader among them.

Since all the agents are equal, it suffices to focus on one agent. We seek a continuous feedback-law $\vec{u_i}$ for the i^{th} agent to achieve multi-agent coordination with obstacle avoidance. More specifically, we seek a barrier-function based controller for the system. We define the barrier function for the i^{th} agent as follows:

$$B_i(\vec{x_i}, \vec{x_j}) = \frac{\|\vec{x_i} - \vec{\tau_i}\|^2}{\|\vec{x_i} - \vec{x_j}\| - d_c + \frac{1}{\epsilon}}$$

where $\epsilon \gg 1$ is a very large number. We define the controller as follows:

$$\vec{v_i} = \begin{cases} -k_1 \|\nabla_{x_i} B_i\|^{\alpha-1} \nabla_{x_i} B_i + (1 - \frac{2(\vec{x_i} - \vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j} & \vec{x} \neq \vec{\tau_i} \\ \vec{0} & \vec{x} = \vec{\tau_i} \end{cases}$$

where $k_1 > 0$ and $0 < \alpha < 1$

With this controller, we have the following result:

Theorem 1: Under the control law, the point $\vec{x} = \vec{\tau_i}$ is FTS equilibrium for the system and the agent will remain collision avoidance w.r.t any other agents.

Before presenting the proof, we present some useful Lemmas:

*Lemma 1:*Under the control law, the point $\vec{x} = \vec{\tau_i}$ is an equilibrium for the system, *i.e.*,

$$\lim_{\vec{x_i} \to \vec{\tau_i}} -k_1 \|\nabla_{x_i} B_i\|^{\alpha-1} \nabla_{x_i} B_i + (1 - \frac{2(\vec{x_i} - \vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j} = \vec{0}$$

Proof: Consider:

$$\frac{(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{(\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}} = \frac{[2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} - \frac{\|\vec{x_i}-\vec{\tau_i}\|^2}{x_0} (\vec{x_i}-\vec{x_j})^{\mathrm{T}}] \cdot \vec{v_j}}{(\vec{x_i}-\vec{\tau_i}) \cdot \vec{v_j}}$$
$$= 2\text{-}\ \|\vec{x_i} - \vec{\tau_i}\|^2 \frac{}{x_0 \frac{(\vec{x_i}-\vec{x_j}) \cdot \vec{v_j}}{(\vec{x_i}-\vec{\tau_i}) \cdot \vec{v_j}}}$$
$$= 2\text{-}\|\vec{x_i} - \vec{\tau_i}\| \cdot \frac{(\vec{x_i}-\vec{x_j}) \cdot \vec{v_j}}{x_0 \|\vec{v_j}\| \cos\theta}$$

Therefore,

$$\lim_{\vec{x_i} \Longrightarrow \vec{\tau_i}} \frac{(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{(\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}} = 2 \Longrightarrow$$

$$\lim_{\vec{x_i} \Rightarrow \vec{\tau_i}} \frac{(\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}{(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}} = \frac{1}{2} \Rightarrow$$

$$\lim_{\vec{x_i} \Rightarrow \vec{\tau_i}} (1 - \frac{2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j} = \vec{0}$$

Since $\nabla_{x_i} B_i(x_i) = 0$, $\lim_{\vec{x_i} \Rightarrow \vec{\tau_i}} -k_1 \|\nabla_{x_i} B_i\|^{\alpha-1} \nabla_{x_i} B_i = 0$
which leads to

$$\lim_{\vec{x_i} \to \vec{\tau_i}} -k_1 \|\nabla_{x_i} B_i\|^{\alpha-1} \nabla_{x_i} B_i + (1 - \frac{2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j} = \vec{0}$$

Lemma 2: Time derivative of the barrier function $\dot{B}_i$ satisfies:

$$\dot{B}_i = -k_1 \|\nabla_{x_i} B_i\|^{\alpha+1}$$

Proof:

$$\dot{B}_i = (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_i} + (\nabla_{x_j} B_i)^{\mathrm{T}} \cdot \vec{v_j}$$

$$= (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot [-k_1 \|\nabla_{x_i} B_i\|^{\alpha-1} \nabla_{x_i} B_i + (1 - \tfrac{2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j}] + \tfrac{\|\vec{x_i}-\vec{\tau_i}\|^2}{x_0 \|\vec{x_i}-\vec{x_j}\|} \cdot (\vec{x_i}-\vec{x_j})^{\mathrm{T}} \cdot \vec{v_j}$$

$$= -k_1 \|\nabla_{x_i} B_i\|^{\alpha+1} + (2 \cdot \tfrac{\vec{x_i}-\vec{\tau_i}}{x_0} - \tfrac{\|\vec{x_i}-\vec{\tau_i}\|^2}{x_0^2} \cdot \tfrac{\vec{x_i}-\vec{x_j}}{\|\vec{x_i}-\vec{x_j}\|}) \cdot [(1 - \tfrac{2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}}) \vec{v_j}] + \tfrac{\|\vec{x_i}-\vec{\tau_i}\|^2}{x_0 \|\vec{x_i}-\vec{x_j}\|} \cdot$$
$$(\vec{x_i}-\vec{x_j})^{\mathrm{T}} \cdot \vec{v_j}$$

$$= -k_1 \|\nabla_{x_i} B_i\|^{\alpha+1} + 2 \cdot \tfrac{(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0} - (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j} \cdot \tfrac{2(\vec{x_i}-\vec{\tau_i})^{\mathrm{T}} \cdot \vec{v_j}}{x_0 (\nabla_{x_i} B_i)^{\mathrm{T}} \cdot \vec{v_j}})$$

$$= -k_1 \|\nabla_{x_i} B_i\|^{\alpha+1}$$

Lemma 3: In the domain $\mathbf{D_0} = \{\mathbf{x} | \|\vec{x} - \vec{x_j}\| > d_c\}, B(x) \leq \epsilon \|x - \tau_i\|^2$

Proof:

$$\begin{aligned} \|x - x_j\| \geq d_c &\Longrightarrow \|x - x_j\| - d_c \geq 0 \\ &\Longrightarrow \|x - x_j\| - d_c + \tfrac{1}{\epsilon} \geq \tfrac{1}{\epsilon} \\ &\Longrightarrow \tfrac{1}{\|x-x_j\| - d_c + \frac{1}{\epsilon}} \leq \epsilon \\ &\Longrightarrow B_i = \tfrac{\|x-\tau_i\|^2}{\|x-x_j\| - d_c + \frac{1}{\epsilon}} \leq \epsilon \|x - \tau_i\|^2 \end{aligned}$$

Lemma 4: $\nabla_{x_i} B_i$ is non-zero everywhere except the equilibrium point $\vec{\tau_i}$ and the point

$$\vec{x} = \vec{\tau_i} + 2 \tfrac{\|\vec{x_j}-\vec{\tau_i}\| + d_c - \frac{1}{\epsilon}}{\|\vec{x_j}-\vec{\tau_i}\|} (\vec{x_j} - \vec{\tau_i})$$

Proof:

Denote $\nabla_{x_i} B_i = \frac{\partial B_i}{\partial \vec{x_i}}, \nabla_{x_j} B_i = \frac{\partial B_i}{\partial \vec{x_j}}, and x_0 = \|\vec{x_i} - \vec{x_j}\| - d_c + \frac{1}{\epsilon}$

$$\nabla_{x_i} B_i = 2 \tfrac{\vec{x}-\vec{\tau_i}}{x_0} - \tfrac{\|\vec{x}-\vec{\tau_i}\|^2}{x_0^2} \tfrac{\vec{x}-\vec{o}}{\|\vec{x}-\vec{o}\|}$$

Solve the equation gives:

$$x = \tau_i + 2 \tfrac{\|x_j-\tau_i\| + d_c - \frac{1}{\epsilon}}{\|x_j-\tau_i\|} (x_j - \tau_i)$$

or

$$x = x_j + 2 \tfrac{\|\tau_i-x_j\| + d_c - \frac{1}{\epsilon}}{\|\tau_i-x_j\|} (\tau_i - x_j)$$

Lemma 5: In any closed, compact domain $D \subset \mathbf{R^n}$ containing point τ_i and excluding the region $\bar{D} = \{x | \|x - [x_j + 2\frac{\|\tau_i - x_j\| + d_c - \frac{1}{\epsilon}}{\|\tau_i - x_j\|}(\tau_i - x_j)]\| < r, \|x - [x_j + 2\frac{\|\tau_i - x_j\| + d_c - \frac{1}{\epsilon}}{\|\tau_i - x_j\|}(\tau_i - x_j)]\| < r\}$,where r is an arbitrary small positive number, the gradient of the barrier function satisfies:

$$\|\nabla_{x_i} B_i\| \geq c\|x - \tau_i\|$$

where c>0

Proof: It can be easily verified that $\nabla_{x_i} B_i(\tau_i) = 0$. Choose $D_1 = \{x \mid \|x - \tau_i\| < \Delta\}$, where Δ is a very small positive number. Choose domain $D = D \backslash D_1$ Recall that D doesn't include the ray $\bar{D}$, so $\tilde{D}$ does not include the point as in Lemma 4 . Hence, from Lemma 4 , at any point $x \in \tilde{D}, \nabla_{x_i} B_i \neq 0$ and since $\tilde{D}$ is a closed domain, we can find $c_1 = \min_{\vec{x} \in \tilde{D}} \frac{\|\nabla_{x_i} B_i\|}{\|\vec{x} - \tau\|} > 0$. Therefore,we have that $\forall x \in \tilde{D}, \|\nabla_{x_i} B_i\| \geq c_1\|x - \tau_i\|$

Next, consider $D_2 = \{x \mid \|x - \tau_i\| \leq \Delta\}$. In a very small neighborhood of τ_i , the Hessian matrix $\nabla^2 B_{ix_i} \succ 0$ (i.e. $\nabla^2 B_{ix_i}$ is a positive definite matrix). Therefore, using the gradient inequality (First-order condition for convexity), we have that $\forall x \in D_2$,

$$\begin{aligned}
&B_i(\vec{\tau_i}) \geq B_i + (\nabla_{x_i} B_i)^T(\vec{\tau_i} - \vec{x}) \\
&\Longrightarrow 0 \geq B_i - (\nabla_{x_i} B_i)^T(\vec{x} - \vec{\tau_i}) \\
&\Longrightarrow (\nabla_{x_i} B_i)^T(\vec{x} - \vec{\tau_i}) \geq B_i
\end{aligned}$$

It is obvious that B_i can be bounded as $B_i \geq c_2\|\vec{x} - \vec{\tau_i}\|^2$. Also, using Cauchy-Schwartz inequality, we have that $(\nabla_{x_i} B_i)^T(\vec{x} - \vec{\tau_i}) \leq \|\nabla_{x_i} B_i\|\|\vec{x} - \vec{\tau_i}\|$ Therefore, we have that

$$\begin{aligned}
\|\nabla_{x_i} B_i\|\|\vec{x} - \vec{\tau_i}\| &\geq (\nabla_{x_i} B_i)^T(\vec{x} - \vec{\tau_i}) \\
&\geq B_i \geq c_2\|\vec{x} - \vec{\tau_i}\|^2 \\
\Longrightarrow \|\nabla_{x_i} B_i\| &\geq c_2\|\vec{x} - \vec{\tau_i}\|
\end{aligned}$$

Now we are ready to prove Theorem 1:

Proof: From Lemma 4, we have that $\nabla_{x_i} B_i = \mathbf{0}$ at the equilibrium point τ_i and at the point $x = \tau_i + \mu(o - \tau_i)$ where μ takes the value as per Lemma 3 . Lets assume that the initial condition is such that $x(t_0)$ doesn't lie on the ray $\bar{D}$ defined as per Lemma 4. Consider the open domain around the goal location D_0 as defined in Lemma 2 . Define $\mathcal{D} = D_0 \backslash \bar{D}$ since $\bar{D}$ is a closed domain and D_0 is open, domain $\mathcal{D}$ is an open domain around the equilibrium τ_i.
Choose the candidate Lyapunov function

$$V_i = B_i$$

From Lemma 2 we have:

$$\dot{V_i} = -k_1\|\nabla_{x_i} B_i\|^{\alpha+1}$$

From Lemma 5 we have:

$$\|\nabla_{x_i} B_i\| \geq c_0 \|\vec{x} - \vec{\tau_i}\| \Longrightarrow \|\vec{x} - \vec{\tau_i}\| \leq \frac{\|\nabla_{x_i} B_i\|}{c_0}$$

From Lemma 3 we have:

$$\begin{gathered} V_i = B_i \leq \epsilon \|\vec{x} - \vec{\tau_i}\|^2 \leq \epsilon \cdot \frac{\|\nabla_{x_i} B_i\|^2}{c_0^2} \\ \Longrightarrow \|\nabla_{x_i} B_i\|^2 \geq \frac{c_0^2}{\epsilon} B_i \end{gathered}$$

Therefore, we have:

$$\dot{V_i} = -k_1 \|\nabla_{x_i} B_i\|^{\alpha+1} \leq -k_1 \frac{c_0^{\alpha+1}}{\epsilon^{\alpha+1}} (B_i)^{\frac{\alpha+1}{2}} = -k_1 \frac{c_0^{\alpha+1}}{\epsilon^{\alpha+1}} (V_i)^{\frac{\alpha+1}{2}}$$

If we set $k_1 \frac{c_0^{\alpha+1}}{\epsilon^{\frac{\alpha+1}{2}}} = c > 0$ and $0 < \frac{\alpha+1}{2} = \beta < 1$, then

$$\dot{V_i} \leq -c V_i^{\beta}$$

which satisfies the condition for FTS.

Chapter 5

Simulation

To test out the efficacy of our designed controller, we demonstrated the whole process using computer simulation. Simulation mainly tested one simple case, one complex case randomly chosen and one arranged as a letter.

For all three cases, the radius of agents is $r = 0.99$, the radius of total move area $R = 98$ (large enough R is one of the keys to success).

In the barrier function:

$$B_i = \frac{\|\vec{x_i} - \vec{\tau_i}\|^2}{\|\vec{x_i} - \vec{x_j}\| - d_c + \frac{1}{\epsilon}}$$

We chose $d_c = 2$, $\epsilon = 10000$ for all three cases. d_c should be slightly larger than $2r$, and ϵ much larger than 1. Too large d_c or too small ϵ will cause agents never get close even when no collision will occur.

In the controller:

$$v_i = -k_1\|\nabla_{x_i} B_i\|^{\alpha-1}\nabla_{x_i} B_i + \dots$$

We chose $\alpha = \frac{1}{3}$, and k_1 differ from agents, magnitude in 10^3.

2000 obstacles are arranged on the edge evenly, ensuring no agents can cross it. The radius of obstacles is the same as agents. In other words, obstacle can be directly viewed as still agents (velocity is strictly 0).

Besides that, in discrete simulation the whole process is divided into 2500 status, every interval is 10^{-4}s.

In simple case, 4 agents are randomly set the original positions and goal position, only ensure that two goal positions are not too close. And in complex case, the number of agents increased to 20. They can all reach the goal in 2500 status. At last, we artificially select goal positions, arrange them in the shape of an H or an A. They can reach the goal successfully too.

For the simulation, most cases tested are successful, only few ones will meet some hard situation that 1 or 2 agents are stuck. Nevertheless, the efficacy of this controller is satisfactory overall.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this paper, we have investigated the distributed coordination problem with collision avoidance in finite time. The idea is inspired by [10]; yet,we developed a controller for multiple agents.However, the drawback of designed controller is that there are some trivial equilibrium points. Considering the complexity to discuss these points, we adopt simplicity. And we simplify the discussion of the points where the denominator stands at zero.

6.2 Future work

1)Developing a second-order control law for the multi-agent system,

2)Discussing the situation where the destinations of the agents are dynamic,

3)Trying different and maybe more strict set-up, for example, agents' sensing and connection areas are constricted.

Bibliography

[1] Sanjay P Bhat and Dennis S Bernstein. Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, 38(3):751–766, 2000.

[2] JF Flores-Resendiz, E Aranda-Bricaire, J González-Sierra, and J Santiaguillo-Salinas. Finite-time formation control without collisions for multiagent systems with communication graphs composed of cyclic paths. *Mathematical Problems in Engineering*, 2015, 2015.

[3] Kunal Garg and Dimitra Panagou. New results on finite-time stability: Geometric conditions and finite-time controllers. In *2018 Annual American Control Conference (ACC)*, pages 442–447. IEEE, 2018.

[4] Meng Ji and Magnus Egerstedt. Distributed coordination control of multiagent systems while preserving connectedness. *IEEE Transactions on Robotics*, 23(4):693–703, 2007.

[5] Dimitra Panagou, Dušan M Stipanović, and Petros G Voulgaris. Distributed coordination control for multi-robot networks using lyapunov-like barrier functions. *IEEE Transactions on Automatic Control*, 61(3):617–632, 2015.

[6] Housheng Su, Xiaofan Wang, and Guanrong Chen. Rendezvous of multiple mobile agents with preserved network connectivity. *Systems & Control Letters*, 59(5):313–322, 2010.

[7] Long Wang and Feng Xiao. Finite-time consensus problems for networks of dynamic agents. *IEEE Transactions on Automatic Control*, 55(4):950–955, 2010.

[8] Xiaoli Wang and Yiguang Hong. Finite-time consensus for multi-agent networks with second-order agent dynamics. *IFAC Proceedings volumes*, 41(2):15185–15190, 2008.

[9] Michael M Zavlanos, Ali Jadbabaie, and George J Pappas. Flocking while preserving network connectivity. In *2007 46th IEEE Conference on Decision and Control*, pages 2919–2924. IEEE, 2007.

[10] Zongyu Zuo and Lin Tie. A new class of finite-time nonlinear consensus protocols for multi-agent systems. *International Journal of Control*, 87(2):363–370, 2014.